and a is the central amplitude. It may be pointed out here that, in all cases, for an n element idealization, the frequency ratios are calculated by dividing ω_{NL} by ω_L, both obtained with the same element idealization.

Tables 1 and 2 present the convergence study for simply supported and clamped beams, respectively, for $l/r=100$ and 10, and for $a/r=1.0$, 2.0, and 3.0. It is seen from these tables that, for a simply supported beam an 8-element idealization, and for a clamped beam a 16-element idealization in the half of the beam, gave satisfactory convergence of the ratio ω_{NL}/ω_L. (The rate of convergence can be improved further by assuming a cubic polynomial for γ, also in the displacement distribution.) Hence, for all further calculations, these element idealizations are used.

Tables 3 and 4 give the ratios of ω_{NL}/ω_L of simply supported and clamped beams, respectively, for various l/r and a/r ratios. From these tables it is observed that the effect of shear deformation and rotatory inertia is to increase the nonlinearity, and this effect increases as l/r decreases. For slender beams (for example $l/r=100$), the results obtained by the present formulation agree well with the results, which also are included in these tables, reported in Ref. 1-3.

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An Experiment on the Lift of an Accelerated Airfoil

G.T. Minkkinen*, T.A. Wilson†, and G.S. Beavers‡
University of Minnesota, Minneapolis, Minn.

Introduction

THIS Note presents the results of an experiment to study the lift characteristics of a two-dimensional airfoil undergoing very high accelerations. The experimental data were obtained by mounting a stationary airfoil in the driver section of a simple shock tube, and utilizing the high fluid accelerations which occur in an expansion fan. In performing the experiment with the airfoil stationary and the gas accelerating, we made use of this result: that the flow and forces which act on a stationary airfoil in an accelerating flow are the same as those on an accelerating airfoil, except for the bouyancy force due to the fluid acceleration which is small, and is orthogonal to the lift. This equivalence between unsteady motion of a body and unsteady motion of a fluid stream about a stationary body appears to be made quite frequently in unsteady flow problems.[1]

The motivation for this experiment arose from work associated with the flight of small insects, which appear to require anomalously high values of the lift coefficient to sustain flight. In discussing this effect, Osborne[2] concluded that the large lift coefficients obtained for insect flight must be attributed to acceleration effects. Recently Weis-Fogh[3] has indicated that nonsteady aerodynamics must play a major role in the flight of very small insects. We have attempted to display the influence of acceleration effects in Fig. 1, which shows a plot of minimum lift coefficient required to support the insect weight plotted against a parameter $\dot{U}c/U^2$, which we call the relative acceleration parameter. These data were taken from Osborne's data for 25 different insects. The velocity U was taken as the vector sum of the flight velocity and the flapping vel., and both U and $\dot{U}$ were computed at a radius equal to 0.75 of the wing length, assuming sinusoidal motion of the wing. The chord length c was computed from the given wing area and length assuming rectangular shape. The final values of $\dot{U}c/U^2$ were computed from an average of 10 positions over a quarter cycle taken from the top of the stroke.

It can be seen from Fig. 1 that the insect lift coefficient increases with increasing values of $\dot{U}c/U^2$. This led us to consider the simple experiment described herein, in which the effect of airfoil acceleration on lift is isolated and evaluated. The experiments were performed at comparable values of $\dot{U}c/U^2$ to those shown for insects in Fig. 1. However, it is clear that our experiment does not model insect flight, since the particular time history of the flapping motion is not reproduced. Furthermore, other features of insect wings such as wing shape, surface properties, wing rotation and bending, all of which have been suggested as possible explanations of high lift, are not included in our experiment.

Experiments

The experiments were performed in a shock tube of square cross section with an internal side length of 3 in. Air was used for both the high and low pressure gases. All unsteady pressures were recorded as functions of time, using Kistler model 601A pressure transducers and model 566 charge amplifiers.

The airfoil model was a NACA 0015 symmetric profile with a span of 3.0 in. and a chord of 0.5 in. Five identical airfoils were manufactured, each mounted at a fixed angle of attack (10°, 14°, 25°, 30°,40°) to a short cylindrical sting. The airfoils were mounted so that positive lift forces were downwards. The sting was placed through a linear ball bushing mounted on the bottom of the shock tube, and was fitted with a collar to prevent motion out of the vertical direction. The end of the sting rested on a thin nylon cap, which in turn rested on the head of a Kistler pressure transducer. With this arrangement, lift forces were recorded in the same way as pressure forces. All force and pressure measurements were made at a location 1 ft from the diaphragm position.

The output of the lift-measuring transducer was corrected to account for the fact that part of the transducer diaphragm was exposed to the local pressure in the rarefaction wave. Correction curves were obtained by running experiments using the sting without an attached airfoil, for initial diaphragm pressure ratios, P, of 2.0, 2.5 and 3.0. These 3 diaphragm pressure ratios were used for all the experiments with the airfoils.

Steady-state values of the lift coefficient were measured in order to provide a reference for the unsteady flow values. This was accomplished by opening both ends of the shock tube and installing a small blower at one end.

Results

The gas particle velocity in a simple rarefaction wave is given in terms of the local pressure by

$$U=\frac{2a_o}{\gamma-1}\left[1-(p/p_o)\exp(\gamma-1)/2\gamma\right] \tag{1}$$

where p_o and a_0 are, respectively, the pressure and speed of sound in the initially undisturbed high pressure gas. The par-

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*Graduate Student, Dept. of Aerospace Engineering & Mechanics.

†Professor, Department of Aerospace Engineering & Mechanics.

‡Professor, Department of Aerospace Engineering & Mechanics. Member AIAA.

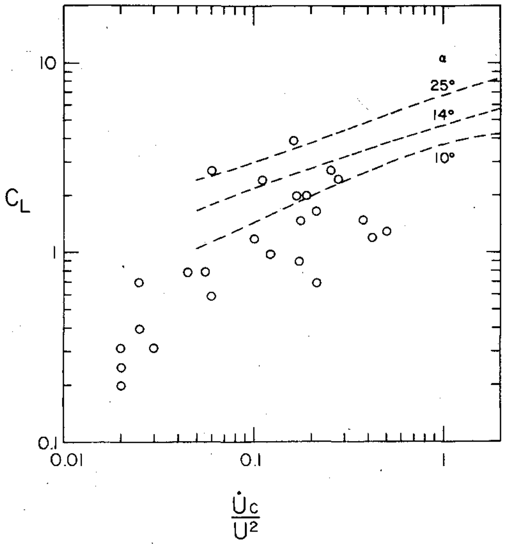

Fig. 1 Lift coefficient as a function of relative acceleration parameter for 25 different insects (derived from the data of Osborne[2]).

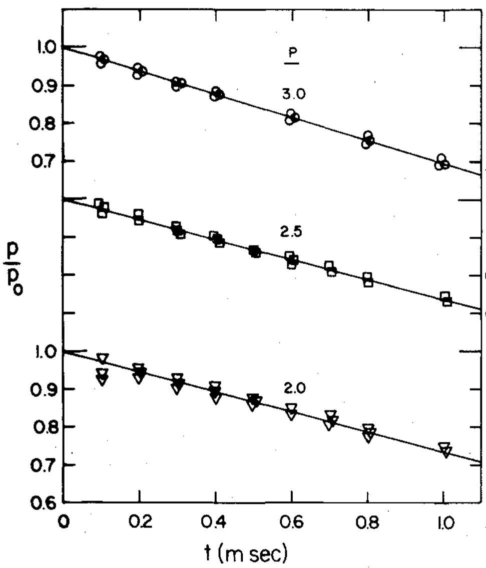

Fig. 2 Pressure at the measuring position in the shock tube as a function of time.

ticle acceleration is then

$$\dot{U} = -\frac{a_0}{\gamma}\left(\frac{p}{p_0}\right) - \exp\ (\gamma+1)/2\gamma \frac{d}{dt}\left(\frac{p}{p_0}\right) \tag{2}$$

Thus the velocity and acceleration at any position are obtained at once as functions of time from the measured time variation of the pressure at that point, and ideally a large range of values of $\dot{U}c/U^2$ can be obtained from a single experimental run.

It was experimentally more convenient to measure the pressure variation with time at the airfoil position before measuring the lift force. Pressure variations were recorded several times for each of the initial diaphragm pressure ratio values of 2.0, 2.5, and 3.0, and are shown in Fig. 2. These experiments indicate that the pressure at the pressure measuring station decreased linearly with time during the early part of the motion, to within the accuracy of the experimental measurements.

Lift measurements were made at angles of attack of 10°, 14°, 25°, and 30°. At an angle of attack of 40°, it was never possible to record a lift force, and it was deduced that the airfoil was in a stalled situation. Useful data could not always be

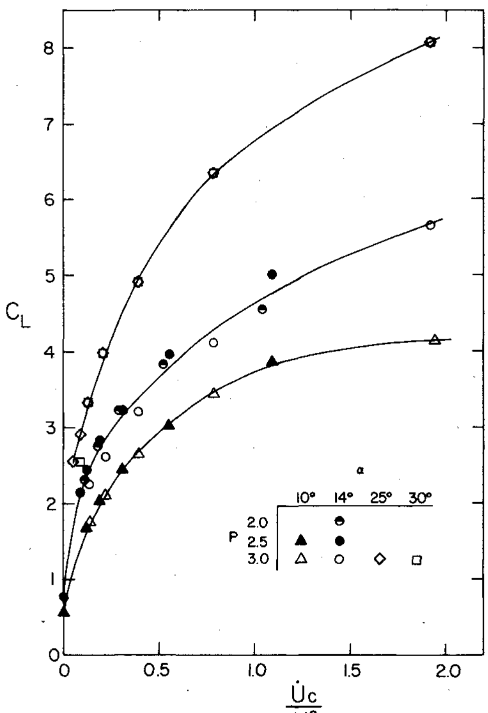

Fig. 3 Lift coefficient as a function of relative acceleration parameter for a symmetric airfoil model at various angles of attack.

obtained for all angles of attack at all 3 of the initial diaphragm pressure ratios. In particular at the 2 highest angles of attack it was only possible to record a lift force for the highest (3.0) initial diaphragm pressure ratio. Where possible, several experimental runs were carried out for the same operating conditions in an attempt to minimize the errors involved in interpreting the output from the transducer.

The unsteady lift coefficients are shown as functions of the acceleration parameter in Fig. 3, from which several observations can be made. First, there is a significant increase in the lift coefficient for increasing values of the acceleration parameter. Next, for a given angle of attack the results appear to be independent of the initial diaphragm pressure ratio, as would be expected if the lift coefficient is only a function of the acceleration parameter. Finally, it is noted that the stall angle of the airfoil was much greater in the accelerating flow than under steady flow conditions. For steady flows it was found that the airfoil profile used for these experiments stalled at about 17°, whereas in the unsteady flow experiments in the shock tube positive lift was obtained for an angle of attack of 30°.

It is interesting to note the exceedingly large lift coefficients at the highest values of the parameter $\dot{U}c/U^2$. For decreasing values of $\dot{U}c/U^2$, the unsteady data appear to approach the values obtained under steady flow conditions for unstalled angles of attack. The curves of lift coefficient vs acceleration parameter for the two angles of attack, 10° and 14°, for which the airfoil was not stalled in steady flow can be reduced to a single curve by normalizing the lift coefficients on the value of the lift coefficient for steady flow at that angle of attack.

Another point to note is that the measured lift coefficients are in approximately the same range as the insect lift coefficients for the same range of $\dot{U}c/U^2$ values. This is indicated in Fig. 1, in which the data of Fig. 3 have been shown as dashed lines. In view of the aforementioned major differences between the insect wing, it would be difficult to make any definitive statement concerning insect flight based on this experiment. However, the fact that an airfoil exhibits high lift coefficients when subjected to rapidly accelerating flows, suggests that wing acceleration may possibly contribute in part to the generation of the high lift coefficients, which some insects require to sustain flight.

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Response of a Symmetric Missile in a Spin-Varying Environment

Ben Zion M. Naveh*

Israeli Government, Tel Aviv, Israel

Nomenclature

$A_{1,2}$	= constants
a_n	= coefficients in a power series expansion
b	$= (M_q\tau_\infty/I + i\omega_0)/c$
c	$= L_p/I_x$
C_D	= drag coefficient
C_{L_α}	= lift coefficient
C_{M_α}	= static moment coefficient
$C_{M_{p\alpha}}$	= Magnus moment coefficient
C_{l_p}	= roll damping moment coefficient
$C_{M_{\dot\alpha}}, C_{M_q}$	= damping moment coefficients
I_x, I	= axial, transversal moments of inertia
i	$=\sqrt{-1}$
K_j	= amplitude of j mode ($j=1,2$)
L_p	$=\rho VS\ell^2 C_{lp}/4$
m	= mass
M_α	$=\rho V^2 S\ell C_{M_\alpha}/2$
M_q	$=\rho VS\ell^2 C_{M_q}/4$
n	= integer
P	$= d\phi/dt$, roll rate
P_l	$= P_\infty - P$
q	= complex angular velocity ($q = i\tilde r + \tilde q$)
$\tilde q, \tilde r$	= angular velocity components
R	$= iI_x/Ic$
r_j	$= [M_q(1\pm\tau_\infty)/2I + i(\eta\pm\omega_0)]/c$
S	= reference area
t	= time
V	= magnitude of velocity
$\alpha_l \beta_l$	= angles of attack and sideslip
α	$=\beta_l + i\alpha_l$
η	$= I_x P/2I$
λ_j	= damping rate of the j mode amplitude $\dot K_j/K_j$
ρ	= air density
ϕ	= roll angle
ϕ_j	= j-modal phase angle
Φ_j	= degenerate hypergeometric functions
τ	$=\eta/\omega$
ω	$=(\omega_0^2+\eta^2)^{1/2}$
ω_0	$=(M_\alpha/I)^{1/2}$

Superscripts

$(\ )'$	= primes denote derivatives with respect to s
$(\dot{\ })$	= dots denote derivatives with respect to t
$(\ )^*$	= stars denote derivatives with respect to P_l

Subscripts

0	= initial value
∞	= value at $t=\infty$
n	= index
e	= envelop

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*Research Engineer. Member AIAA.

Approximate Solution

THE response characteristics of a fin-stabilized symmetric missile for slowly time-varying parameters were formulated by C.H. Murphy.[1] Consider a motion about the center of gravity in which all flight parameters and aerodynamic coefficients are constant, but where the roll rate is varying according to

$$P = P_\infty + (P_0 - P_\infty)e^{ct} \tag{1}$$

where $c<0$. Expressing Murphy's formulations in the time domain

$$\dot\phi_j = \eta \pm \omega \tag{2}$$

$$\dot\eta = (L_P/I_x)(\eta - \eta_\infty) \tag{3}$$

$$\ddot\phi_j = \dot\eta(1\pm\tau) \tag{4}$$

and the damping rates are

$$\lambda_j = \frac{M_q}{2I}(1\pm\tau) \mp \frac{L_p}{2I_x}(\tau-\tau_\infty)(1\pm\tau) \pm \frac{M_{p\alpha}}{I_x}\tau \tag{5}$$

Equation (5) suggests that there might be combinations for which λ_j will become positive for some time interval. Consider a special case, in the absence of Magnus moment.

Decreasing roll rate flight for which $P_\infty=0$, Eq. (5) becomes

$$\lambda_j = \frac{1\pm\tau}{2I}\left(M_q \mp \frac{I}{I_x}L_p\tau\right) \tag{6}$$

For some P_0, λ_l might become positive, for some time interval. Equation (5) should also be considered when test data is analyzed to determine Magnus moment coefficient.

Exact Solution

The finding that λ_j might become positive due to roll damping terms was considered important, and an attempt to find an exact solution was made. Consider the increasing roll rate case ($P_0=0$), with the previous assumptions.

The equation of motion in aeroballistic axes is[2]

$$\ddot\alpha - \left(\frac{M_q}{I} + i\frac{I_x}{I}p\right)\dot\alpha + \omega_0^2\alpha = 0 \tag{7}$$

Change the independent variable from t to P_l

$$P_l = P_\infty e^{ct} \tag{8}$$

and get

$$P_l^2\alpha^{**} + \left[\left(1 - \frac{M_q}{Ic} - i\frac{I_x}{Ic}P_\infty\right)P_l + i\frac{I_x}{Ic}P_l^2\right]\alpha^* + \left(\frac{\omega_0}{c}\right)^2\alpha = 0 \tag{9}$$

α is assumed to be presented in power series of P_l

$$\alpha = P_l^r \sum_{n=0}^{\infty} a_n P_l^n \tag{10}$$